MATH 135: Written Assignment 1

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Problem 1

(1a) The perfect cubes between -10 and 10 are:

$$-8, -1, 0, 1, 8$$

(1b) $P(n) \equiv \exists\, a, b, c, d \in \mathbb{Z}, n = a^3 + b^3 + c^3 + d^3$

- Proving $P(4)$: $4 = 1^3 + 1^3 + 1^3 + 1^3$
- Proving $P(5)$: $5 = 2^3 + (-1)^3 + (-1)^3 + (-1)^3$
- Proving $P(-5)$: $-5 = (-2)^3 + 1^3 + 1^3 + 1^3$

(1c) $\forall\, n \in \mathbb{Z}, \exists\, a, b, c, d \in \mathbb{Z}, n = a^3 + b^3 + c^3 + d^3$

(1d) $\exists\, n \in \mathbb{Z}, \forall\, a, b, c, d \in \mathbb{Z}, n \neq a^3 + b^3 + c^3 + d^3$

Problem 2

(2a)

- $\because s(7) = 1 \neq 7, \therefore 7$ is **not** a perfect number.
- $\because s(14) = 1 + 2 + 7 = 10 \neq 14, \therefore 14$ is **not** a perfect number.
- $\because s(21) = 1 + 3 + 7 = 11 \neq 21, \therefore 21$ is **not** a perfect number.
- $\because s(28) = 1 + 2 + 4 + 7 + 14 = 28, \therefore 28$ **is** a perfect number.

(2b) $\exists\, n \in \mathbb{N}, \forall\, t \in T, t \leq n$

(2c) $\forall\, t \in T, \exists\, k \in \mathbb{N}, t = 2k$

(2c) $\exists\, t \in T, \forall\, k \in \mathbb{N}, t \neq 2k$

Problem 3

(3a) $S = \{1, 2\}$, $P(a, b) : 2a^3 - 7a^2b + 7ab^2 - 2b^3$

(3b) $S = \mathbb{Q}$, $P(a, b) : (a^2 + 2)(b + 3) = 1$

(3c) $S = \mathbb{R}$, $P(a, b) : a + \sqrt{2}$

(3d) $S = \mathbb{Z}$, $P(a, b) : 2^a - 7 = b^2$

Problem 4

(4a) True

(4b) False

(4c) True

(4d) False

(4e) True

(4f) True

(4g) False

(4h) True

Problem 5

(5a)

$Q(1, 10)$:

$$\begin{aligned}\exists\, n \in \mathbb{Z}, R(1, 10, n) \\ \exists\, n \in \mathbb{Z}, 4(1) + 5(10) = 2n^3 \\ \exists\, n \in \mathbb{Z}, 27 = n^3 \\ \exists\, n \in \mathbb{Z}, n = 3 : \textbf{True}\end{aligned}$$

$R(-3, 13, 3)$:

$$\begin{aligned}4(-3) + 5(13) = 2(3)^2 \\ 53 = 18 : \textbf{False}\end{aligned}$$

(5b) This is an open sentence which depends on the value of w.

(5c) This is an open sentence which depends on the value of v.

(5d) This is a *mathematical statement* with value **False**:

$$\begin{aligned}\forall\, v \in \mathbb{Z}, \exists\, k \in \mathbb{Z}, Q(v, 2k + 1) \\ \forall\, v \in \mathbb{Z}, \exists\, k \in \mathbb{Z}, \exists\, n \in \mathbb{Z}, R(v, 2k + 1, n) \\ \forall\, v \in \mathbb{Z}, \exists\, k \in \mathbb{Z}, \exists\, n \in \mathbb{Z}, 4v + 5(2k + 1) = 2n^3 \\ \frac{4v + 5(2k + 1)}{2} = n^3 \\ 2v + \frac{5}{2}(2k + 1) = n^3\end{aligned}$$

Since $\frac{5}{2}(2k + 1)$ is not an integer, the left hand side is also not an integer. However on the right hand side, n is an integer which makes n^3 an integer as well. Since the left side is not an integer, and the right side is an integer, this statement is **false**.

(5e) This is a *mathematical statement* with value **True**:

$$\begin{aligned}\forall\, n \in \mathbb{Z}, \exists\, v, w \in \mathbb{Z}, R(v, w, n) \wedge v, w \neq 0 \\ 4v + 5w = 2n^3 \\ \text{let} v = -w \\ -4w + 5w = 2n^3 \\ w = 2n^3\end{aligned}$$

Case 1: $n = 0$

$$\begin{aligned}\text{let} v = 5, w = -4 \\ 4(5) + 5(-4) = 2(0)^3 \\ 0 = 0 : \text{True}\end{aligned}$$

Case 2: $n \neq 0$

$$w = 2n^3$$

Since $n \in \mathbb{Z}$, $w = 2n^3 \in \mathbb{Z}$ as well. Since $v = -w$, v will also be an integer.